SEQUENCES AND SERIES

1 Expand

a $(1 + 3x)^4$ **b** $(2 - x)^5$ **c** $(3 + 10x^2)^3$ **d** $(a + 2b)^5$

e $(x^2 - y)^3$ **f** $(5 + \frac{1}{2}x)^4$ **g** $(x + \frac{1}{x})^4$ **h** $(t - \frac{2}{t^2})^3$

2 Find the first four terms in the expansion in ascending powers of x of

a $(1 + 3x)^6$ **b** $(1 - \frac{1}{4}x)^8$ **c** $(5 - x)^7$ **d** $(3 + 2x^2)^{10}$

3 Find the coefficient indicated in the following expansions

a $(1 + x)^{15}$, coefficient of x^3 **b** $(1 - 2x)^{12}$, coefficient of x^4

c $(3 + x)^7$, coefficient of x^2 **d** $(2 - y)^{10}$, coefficient of y^5

e $(2 + t^3)^8$, coefficient of t^{15} **f** $(1 - \frac{1}{x})^9$, coefficient of x^{-3}

4 **a** Express $(\sqrt{2} - \sqrt{5})^4$ in the form $a + b\sqrt{10}$, where $a, b \in \mathbb{Z}$.

b Express $(\sqrt{2} + \frac{1}{\sqrt{3}})^3$ in the form $a\sqrt{2} + b\sqrt{3}$, where $a, b \in \mathbb{Q}$.

c Express $(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3$ in the form $a\sqrt{5}$, where $a \in \mathbb{Z}$.

5 **a** Expand $(1 + \frac{x}{2})^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.005^{10} **ii** 0.996^{10}

giving your answers to 5 decimal places.

6 **a** Expand $(3 + x)^8$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 3.001^8 **ii** 2.995^8

giving your answers to 7 significant figures.

7 Expand and simplify

a $(1 + 10x)^4 + (1 - 10x)^4$ **b** $(2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$

c $(1 + 4y)(1 + y)^3$ **d** $(1 - x)(1 + \frac{1}{x})^3$

8 Expand each of the following in ascending powers of x up to and including the term in x^3.

a $(1 + x^2)(1 - 3x)^{10}$ **b** $(1 - 2x)(1 + x)^8$

c $(1 + x + x^2)(1 - x)^6$ **d** $(1 + 3x - x^2)(1 + 2x)^7$

9 Find the term independent of y in each of the following expansions.

a $(y + \frac{1}{y})^8$ **b** $(2y - \frac{1}{2y})^{12}$ **c** $(\frac{1}{y} + y^2)^6$ **d** $(3y - \frac{1}{y^2})^9$

10 The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6

a Find the value of n.

b Use your value of n to find the coefficient of x^4 in the expansion.

11 Given that $y_1 = (1 - 2x)(1 + x)^{10}$ and $y_2 = ax^2 + bx + c$ and that when x is small, y_2 can be used as an approximation for y_1,

a find the values of the constants a, b and c,

b find the percentage error in using y_2 as an approximation for y_1 when $x = 0.2$

12 In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is –12 and the coefficient of x^2 is 60.

Find

a the value of p and the value of q,

b the value of the coefficient of x^3 in the expansion.

13 **a** Expand $(3 - \frac{x}{3})^{12}$ as a binomial series in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

b Use your series expansion with a suitable value of x to obtain an estimate for 2.998^{12}, giving your answer to 2 decimal places.

14 **a** Expand $(1 - x)^5$ as a binomial series in ascending powers of x.

b Express $(\sqrt{3} + 1)(\sqrt{3} - 2)$ in the form $A + B\sqrt{3}$, where $A, B \in \mathbb{Z}$.

c Hence express each of the following in the form $C + D\sqrt{3}$, where $C, D \in \mathbb{Z}$.

i $(\sqrt{3} + 1)^5(\sqrt{3} - 2)^5$

ii $(\sqrt{3} + 1)^6(\sqrt{3} - 2)^5$

15 **a** Expand $(1 + \frac{x}{2})^9$ in ascending powers of x up to and including the term in x^4.

Hence, or otherwise, find

b the coefficient of x^3 in the expansion of $(1 + \frac{x}{2})^9 - (1 - \frac{x}{2})^9$,

c the coefficient of x^4 in the expansion of $(1 + 2x)(1 + \frac{x}{2})^9$.

16 The term independent of x in the expansion of $(x^3 + \frac{a}{x^2})^5$ is –80.

Find the value of the constant a.

17 In the binomial expansion of $(1 + \frac{x}{k})^n$, where k is a non-zero constant, n is an integer and $n > 1$, the coefficient of x^2 is three times the coefficient of x^3.

a Show that $k = n - 2$.

Given also that $n = 7$,

b expand $(1 + \frac{x}{k})^n$ in ascending powers of x up to and including the term in x^4, giving each coefficient as a fraction in its simplest form.